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Unaided EVA Intercept and Rendezvous Charts

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In order to provide a margin of safety for astronauts working untethered in the vicinity of the space shuttle, it is necessary to provide for them a means to determine conditions which would return them to the shuttle via an intercept and rendezvous maneuver. An intercept chart is developed for use in an unaided EVA environment. Given the in-plane azimuth angle, distance away, and time to intercept, the chart provides the direction and magnitude of ΔV required for rendezvous. The chart is valid for nominally circular orbits of all altitudes. Additional charts are presented which provide closing velocity information as well as total ΔV requirements necessary for planning rendezvous maneuvers. Finally, a brief error analysis is presented along with a series of charts which provide a simple method of estimating the effects of errors in initial conditions on terminal position and velocity.

Introduction

IT is likely that in order to take advantage of the space shuttle's capabilities, one or more astronauts will work in the vicinity of the shuttle, but not be tethered to it. Each astronaut will be equipped with a maneuvering unit that will allow him to develop any desired increment of velocity ΔV. For the purposes of safety, it is desirable that the astronaut be able to carry out an intercept and rendezvous maneuver with the shuttle from any relative nearby position. Furthermore, he should be able to complete the maneuver (at least the intercept portion) without reliance on communication with the shuttle. Hence, a minimum requirement is that the astronaut be able to determine the velocity increment and direction required for intercept without aid from the shuttle. A more complete discussion of the problem and its inherent restrictions is given in Ref. 1.

Higgins and Fowler[1] have developed a set of charts useful for determining the required intercept conditions for the case where the astronaut's relative position (i.e., range and azimuth) with respect to the shuttle is known (see Figs. 2 and 3 of Ref. 1). These charts are valid for a specific time to intercept (average closure rate) and for specific shuttle orbit radius (circular orbit is assumed). Consequently, several charts are needed to cover the range of desirable intercept times and the possibility of operating from different nominal orbits. Furthermore, the direction of thrust is difficult to determine with any accuracy from the proposed charts and certainly would be equally as difficult to implement in space, since there is no reference line from which to measure.

The purpose of this paper is to improve upon the charts proposed by Higgins and Fowler in the form of a single chart which is valid for determining intercept conditions for circular orbits of all radii and all intercept times. Furthermore, the chart provides a direct readout of the required ΔV and its direction in terms of average closure rate and angle with respect to the astronaut/shuttle line-of-sight.

In addition, some considerations are given to the problem of rendezvous in the form of a chart for determining the closing velocity and hence the nulling ΔV required, and a chart for determining the total ΔV required for the complete rendezvous procedure. Finally, a brief error analysis is presented and charts are developed which indicate the expected miss distance and azimuth angle, as well as errors in the terminal velocity in terms of initial conditions and errors.

Analysis

Intercept Chart

The development is similar to that in Ref. 1 and assumes two point masses orbiting a spherical Earth. An axis system is selected as shown in Fig. 1 with the origin centered in the shuttle, assumed to be in a circular orbit, the $\tilde{x}$ axis pointing radially outward, the $\tilde{y}$ axis forward in the orbit plane, and the $\tilde{z}$ axis completing the right-hand set. The astronaut will be located initially at some point $(\tilde{x}_0, \tilde{y}_0, \tilde{z}_0)$. The problem then is: given some time of flight, find the initial relative velocity and its direction so that the astronaut will be at the origin of the coordinate system when the selected time of flight is reached.

For the values of initial range of interest here (<2000 m) the linearized analysis of Clohessy and Wiltshire will suffice.[2,3] Should larger initial range distances be considered, the equations of Ref. 3 could be used, or, keeping the equations linear, the little-known improved linear equations of Werlwas[4] could be used. The drawback of the Werlwas equations is that the equation constants depend on initial conditions. In addition, it is convenient to nondimensionalize the equations of motion in the following manner:

Let

$$\tau = \dot{\theta} t \qquad \frac{\mathrm{d}}{\mathrm{d}t} = \dot{\theta}\frac{\mathrm{d}}{\mathrm{d}\tau}$$

$$x = \tilde{x}/\tilde{r}_0 \qquad y = \tilde{y}/\tilde{r}_0 \qquad z = \tilde{z}/\tilde{r}_0$$

where $\dot{\theta}$ is the constant angular rate of the shuttle orbit, and $\tilde{r}_0$ is the initial separation distance between astronaut and shuttle. The resulting nondimensional equations of motion are

$$\begin{aligned} x'' - 2y' - 3x &= 0 \\ y'' + 2x' &= 0 \\ z'' + z &= 0 \end{aligned} \tag{1}$$

where $(\ )' = \mathrm{d}/\mathrm{d}\tau$. Under this linearized development, the out-of-plane motion becomes separated from the in-plane motion and can be treated separately. The procedure for nulling such motion is discussed elsewhere,[1] hence the out-of-plane equation will be dropped from further consideration.

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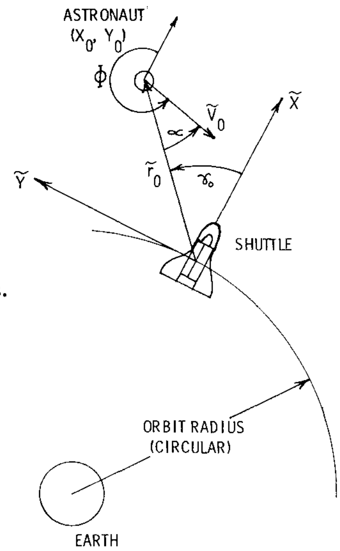

Fig. 1 Geometry of rendezvous.

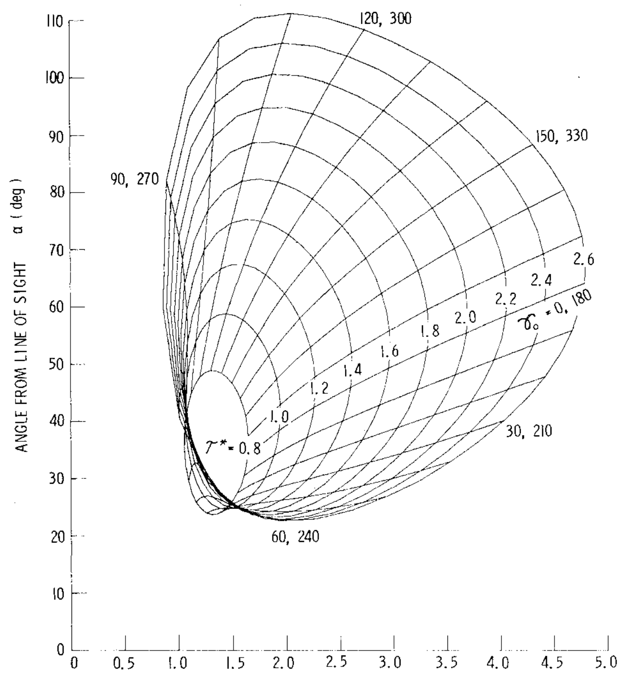

Fig. 2 Nondimensional intercept chart.

The solution to Eq. (1) is given by

$$x = x_0 + (3x_0 + 2y_0')(1-\cos\tau) + x_0'\sin\tau$$

$$y = y_0 - (6x_0 + 3y_0')\tau + (6x_0 + 4y_0')\sin\tau - 2x_0'(1-\cos\tau)$$

$$x' = x_0'\cos\tau + (3x_0 + 2y_0')\sin\tau$$

$$y' = y_0' - (6x_0 + 4y_0')(1-\cos\tau) - 2x_0'\sin\tau \tag{2}$$

For intercept in a nondimensional time τ^* (or transfer angle τ^*)[3] the requirement is simply that $x(\tau^*) = y(\tau^*) = 0$. Hence, for a given initial position x_0, y_0, the preceding equations can be solved for the initial velocity requirement for intercept, leading to the result

$$x_0' = \frac{x_0(3\tau^*\cos\tau^* - 4\sin\tau^*) + 2y_0(1-\cos\tau^*)}{8(1-\cos\tau^*) - 3\tau^*\sin\tau^*}$$

$$y_0' = \frac{2x_0[7(\cos\tau^* - 1) + 3\tau^*\sin\tau^*] - y_0\sin\tau^*}{8(1-\cos\tau^*) - 3\tau^*\sin\tau^*} \tag{3}$$

It is convenient at this time to introduce a nondimensional velocity related to an average closing speed $\tilde{V}_{av} = \tilde{r}_0/t^*$. Hence, define

$$\hat{V} = t^*\tilde{V}/\tilde{r}_0 = \tilde{V}/\tilde{V}_{av} \tag{4}$$

Also, it is easy to show that

$$\hat{V}_{x0} = \tau^* x_0' = \tilde{V}_{x0}/\tilde{V}_{av} \qquad \hat{V}_{y0} = \tau^* y_0' = \tilde{V}_{y0}/\tilde{V}_{av} \tag{5}$$

Furthermore, note that

$$\tilde{x}_0/\tilde{r}_0 = x_0 = \cos\gamma_0 \qquad \tilde{y}_0/\tilde{r}_0 = y_0 = \sin\gamma_0 \tag{6}$$

where γ_0 is the angle measured from the x axis to the relative starting position of the astronaut with respect to the shuttle (azimuth angle, see Fig. 1). In addition, the angle that the required initial velocity for rendezvous makes can be determined from

$$\tan\phi = \tilde{V}_{y0}/\tilde{V}_{x0} = y_0'/x_0' \tag{7}$$

where ϕ is the angle measured from the x axis. If a line-of-sight vector is defined from the astronaut to the shuttle, the angle the required velocity vector makes with the line-of-sight vector can be determined from (see Fig. 1).

$$\alpha = \pi + \phi - \gamma_0 \tag{8}$$

If Eqs. (3-8) are combined in an appropriate manner and, since $\tilde{V}_0 = (\tilde{V}x_0^2 + \tilde{V}y_0^2)^{1/2}$, an expression can be found for the nondimensional velocity required for intercept and its angle from the line-of-sight vector in terms of azimuth angle and nondimensional intercept time. That is,

$$\hat{V}_0 = \tilde{V}_0/\tilde{V}_{av} = f_1(\gamma_0, \tau^*) \qquad \alpha = f_2(\gamma_0, \tau^*) \tag{9}$$

Equation (9) is too cumbersome to express here. However, it is presented graphically as the nondimensional intercept chart given in Fig. 2.

Rendezvous Considerations

In order to turn the preceding interecept maneuver into a rendezvous, it is necessary to null the terminal velocity to zero. This is accomplished by providing a velocity impulse in the opposite direction of the terminal velocity vector, or equivalently, opposite the line-of-sight vector at closure. Hence, the terminal maneuver is easier to implement than the initial maneuver. The terminal velocity for the various intial positions and rendezvous times can be obtained by evaluating Eq. (2) at τ^* with the initial velocity given by Eq. (3). The result, after incorporating Eq. (4) is presented in Fig. 3.

Another consideration when including rendezvous in the problem is that of the total velocity impulse or correspondingly, fuel, required. Such information is necessary when selecting time to rendezvous with a limited amount of fuel available. A chart with this information is presented in Fig. 4.

Error Analysis

During the rendezvous procedure, the astronaut has to estimate his distance and azimuth angle from the shuttle, compute the magnitude and direction of velocity impulse needed, and finally apply the required thrust in the correct direction. A perfect rendezvous would not occur if the astronaut made an error in the magnitude and/or direction of impulse corresponding to his initial position in space. Such an error could come about in three ways: 1) astronaut correctly estimates position and thrusts incorrectly; 2) astronaut incorrectly estimates position and thrusts correctly for

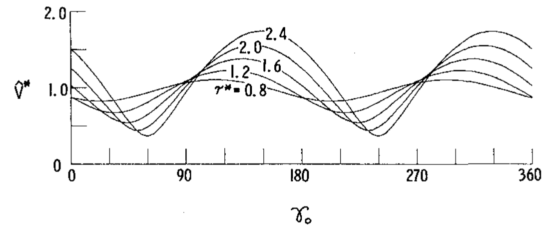

Fig. 3 Terminal velocity chart.

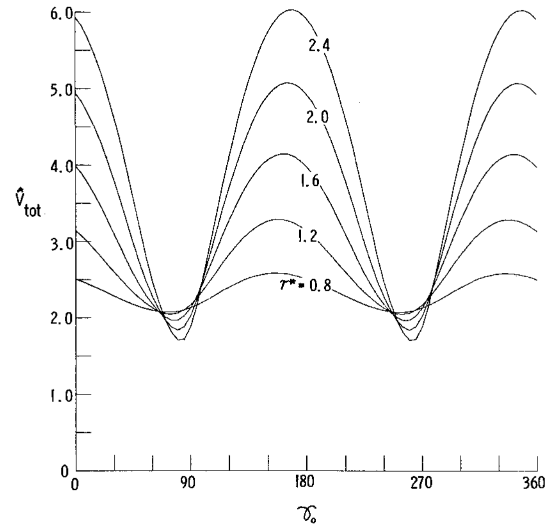

Fig. 4 Total velocity impulse chart.

estimated position; and 3) astronaut incorrectly estimates position and thrusts incorrectly for the estimated position (possibly cancelling the error).

The error to be calculated is that which exists at the prescribed intercept time t^*. (Note that at some time later or earlier the astronaut could possibly be closer to the desired rendezvous point.) Keeping in mind the possible sources of error discussed, the nondimensional error in the x direction at time t^* is given by

$$\Delta x^* = \left(\frac{\partial x^*}{\partial x_0} + \frac{\partial x^*}{\partial \hat{V}_{x0}}\frac{\partial \hat{V}_{x0}}{\partial x_0} + \frac{\partial x^*}{\partial \hat{V}_{y0}}\frac{\partial \hat{V}_{y0}}{\partial x_0}\right)\Delta x_0 + \left(\frac{\partial x^*}{\partial y_0} + \frac{\partial x^*}{\partial \hat{V}_{x0}}\frac{\partial \hat{V}_{x0}}{\partial y_0} + \frac{\partial x^*}{\partial \hat{V}_{y0}}\frac{\partial \hat{V}_{y0}}{\partial y_0}\right)\Delta y_0 + \frac{\partial x^*}{\partial \hat{V}_{x0}}\Delta \hat{V}_{x0} + \frac{\partial x^*}{\partial \hat{V}_{y0}}\Delta \hat{V}_{y0}$$

which can be expressed as

$$\Delta x^* = \frac{Dx^*}{Dx_0}\Delta x_0 + \frac{Dx^*}{Dy_0}\Delta y_0 \frac{\partial x^*}{\partial \hat{V}_{x0}}\Delta \hat{V}_{x0} + \frac{\partial x^*}{\partial \hat{V}_{y0}}\Delta \hat{V}_{y0} \qquad (10)$$

with a similar expression for Δy^*, $\Delta \hat{V}_x{}^*$, and $\Delta \hat{V}_y{}^*$; where $(\ )^*$ means values at $t=t^*$.

In addition the following relations exist:

$$x_0 = \cos\gamma_0 \qquad \Delta x_0 = -\sin\gamma_0 \Delta\gamma_0$$

$$y_0 = \sin\gamma_0 \qquad \Delta y_0 = \cos\gamma_0 \Delta\gamma_0$$

$$\hat{V}_{x0} = \hat{V}_0 \cos\phi \qquad \Delta \hat{V}_{x0} = \Delta \hat{V}_0 \cos\phi - \hat{V}_0 \sin\phi \Delta\phi$$

$$\hat{V}_{y0} = \hat{V}_0 \sin\phi \qquad \Delta \hat{V}_{y0} = \Delta \hat{V}_0 \sin\phi + \hat{V}_0 \cos\phi \Delta\phi$$

and from Eq. (8),

$$\Delta\alpha = \Delta\phi - \Delta\gamma_0 \quad \cos\phi = -\cos(\gamma_0+\alpha) \quad \sin\phi = -\sin(\gamma_0+\alpha)$$

Combining the foregoing, it is possible to determine the errors in x^*, y^*, $\hat{V}_x{}^*$, or $\hat{V}_y{}^*$ in terms of errors in initial azimuth angle, initial velocity magnitude, and initial velocity direction. (Note that because of the definition of the nondimensional length, there can be no error in distance, i.e., $r_0=1$. Clearly an error in estimated range shows up in the assumed average velocity increment $\Delta\hat{V}$.) The result of such a calculation for terminal position is given by

$$\Delta x^* = \left\{\left[\frac{\partial x^*}{\partial \hat{V}_{x0}}\sin(\gamma_0+\alpha) - \frac{\partial x^*}{\partial \hat{V}_{y0}}\cos(\gamma_0+\alpha)\right]\hat{V}_0\right\}(\Delta\alpha+\Delta\gamma_0) - \left[\frac{\partial x^*}{\partial \hat{V}_{x0}}\cos(\gamma_0+\alpha) + \frac{\partial x^*}{\partial \hat{V}_{y0}}\sin(\gamma_0+\alpha)\right]\Delta\hat{V}_0 \qquad (11)$$

with a similar expression for Δy^*. Note that

$$\frac{Dx^*}{Dx_0} = \frac{Dy^*}{Dx_0} = \frac{Dx^*}{Dy_0} = \frac{Dy^*}{Dy_0} = 0$$

Similar calculations for the terminal velocity change result in the expression

$$\Delta\hat{V}_x{}^* = \left\{-\frac{D\hat{V}_x{}^*}{Dx_0}\sin\gamma_0 + \frac{D\hat{V}_x{}^*}{Dy_0}\cos\gamma_0 + \left[\frac{\partial\hat{V}_x{}^*}{\partial\hat{V}_{x0}}\sin(\gamma_0+\alpha) - \frac{\partial\hat{V}_x{}^*}{\partial\hat{V}_{y0}}\cos(\gamma_0+\alpha)\right]\hat{V}_0\right\}\Delta\gamma_0 + \left[\frac{\partial\hat{V}_x{}^*}{\partial\hat{V}_{x0}}\sin(\gamma_0+\alpha) - \frac{\partial\hat{V}_x{}^*}{\partial\hat{V}_{y0}}\cos(\gamma_0+\alpha)\right]\hat{V}_0\Delta\alpha - \left[\frac{\partial\hat{V}_x{}^*}{\partial\hat{V}_{x0}}\cos(\gamma_0+\alpha) + \frac{\partial\hat{V}_x{}^*}{\partial\hat{V}_{y0}}\sin(\gamma_0+\alpha)\right]\Delta\hat{V}_0 \qquad (12)$$

with a similar expression for $\Delta V_y{}^*$.

The derivatives necessary for evaluating Eqs. (11) and (12) can be determined from Eqs. (2), (3), and the definition in Eq. (4). The terms in the brackets can thus be evaluated in terms of the initial azimuth angle γ_0 and rendezvous angle τ^*. The coefficients in Eq. (11) are plotted in Figs. 5 and 6, while those in Eq. (12) are plotted in Figs. 7-9.

Discussion

Intercept Chart

The nondimensional intercept chart in Fig. 2 is obtained by plotting the functions represented by Eq. (9) for a range of values of constant azimuth angle, γ_0, and transfer angle (time), τ^*. The result is a chart which is valid for all reasonable orbit radii and intercept times.

In order to use the chart, the astronaut would estimate his distance and azimuth angle with respect to the shuttle and then select a desired rendezvous time. Once the time is selected, the transfer angle is obtained by multiplying by the orbital mean angular rate. It is assumed that this information would be available to the astronaut prior to his EVA. More likely, however, the astronaut will be furnished with desired values of transfer angles corresponding to a typical range of intecept times. Given the time and the estimated distance, the average closing velocity $\hat{V}_{av}$ can be determined. From the chart at the intersection of the line of (constant) azimuth angle γ_0 and line of (constant) transfer angle τ^*, the nondimensional initial velocity $\hat{V}$ and direction α can be read

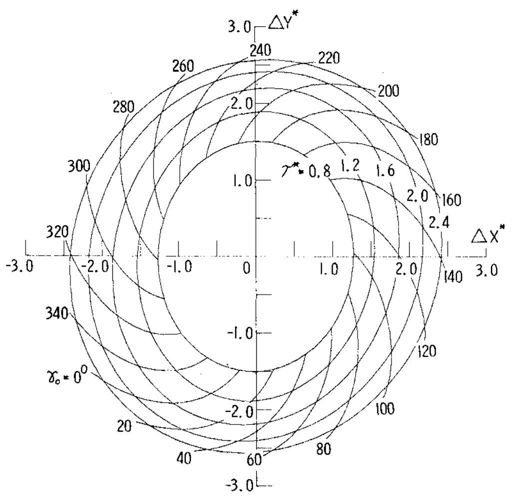

Fig. 5 Terminal position error due to unit error in thrust angle α or azimuth angle γ_0.

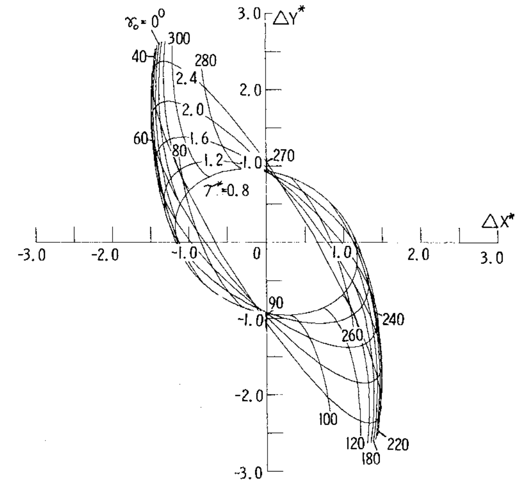

Fig. 6 Terminal position error due to unit error in initial velocity $\hat{V}_0$.

from the abscissa and the ordinate, respectively. The actual initial velocity can be determined by multiplying $\hat{V}$ by $\tilde{V}_{av}$.

Although more calculations are required here than when using the charts of Higgins and Fowler, it is felt that the accuracy obtained in determining the direction of V_0 is worth it. Basically, only one multiplication and division are needed, operations available even on the smallest of calculators. Furthermore, only one chart is required and the requirements for an intercept from a given relative position for various intercept times could be investigated if desired.

If the astronaut has an initial relative drift velocity he must be able to estimate it and have the capability of taking the vector difference between the desired intercept velocity and his current drift velocity to obtain the required $\Delta\hat{V}$ and α for intercept. Again, the calculations could be easily done on a small calculator or on a special slide rule similar to those pilots use to determine ground speed in the presence of crosswinds. The estimation of drift speed and direction is a little more difficult even with accurate range and range rate data. Consequently, this estimate is a source of considerable error and needs additional investigation.

Rendezvous Considerations

Figure 3 shows the terminal velocities which must be nulled if a "soft landing" or rendezvous is to be obtained. The purpose of this chart is to indicate the order of magnitude of the terminal velocities encountered for the range of initial

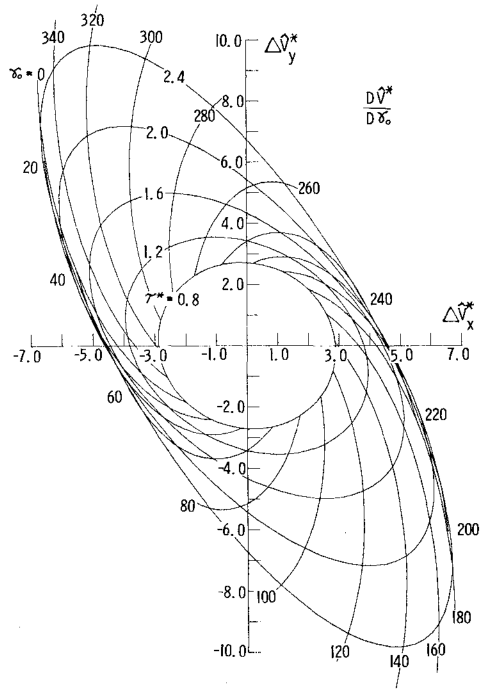

Fig. 7 Terminal velocity error due to unit error in azimuth angle γ_0.

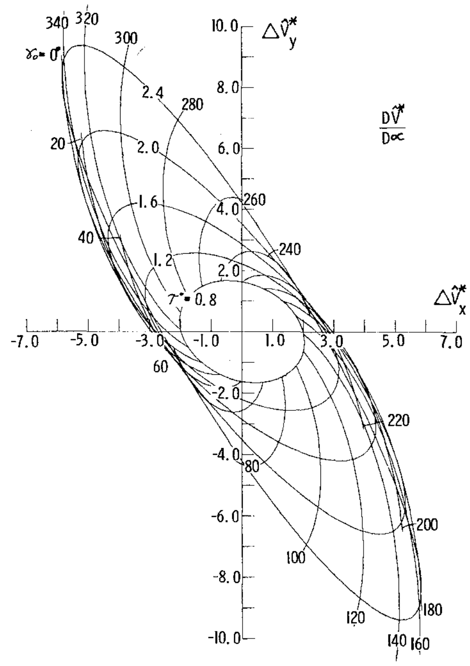

Fig. 8 Terminal velocity error due to unit error in thrust angle α.

conditions considered. In no case does the terminal speed exceed 1.8 $\hat{V}_{av}$. The astronaut would provide the velocity impulse, $\Delta\hat{V}^* = \hat{V}^*$, corresponding to his starting position and rendezvous time, in the opposite direction of his line-of-sight at closure.

At the start of the intercept maneuver the information provided by Fig. 4 would be of considerable use to the astronaut. It shows the total $\Delta\hat{V}$ (fuel) required for the complete rendezvous maneuver for given initial position and rendezvous time. Consequently, the astronaut has the information necessary to consider fuel tradeoffs with rendezvous time in case his fuel reserves are critically low.

For example consider a contemplated rendezvous from a position where $\bar{r}_0 = 1000$ m and $\gamma_0 = 0$ deg. If the time to

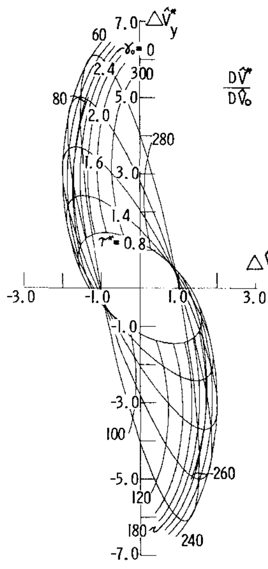

Fig. 9 Terminal velocity error due to unit error in initial velocity $\hat{V}_0$.

rendezvous is selected to be 1000 s with the corresponding transfer angle $\tau^*=0.8$ rad, the total $\Delta\hat{V}$ required is obtained from Fig. 4 to be $\Delta\hat{V}_{\text{tot}}=2.5\ (V_{\text{av}})=2.5\cdot(1\text{m/s})=2.5$ m/s. If the rendezvous time were tripled, $\tau^*=2.4$ rad and the total $\Delta\hat{V}$ required is reduced to $\Delta\hat{V}_{\text{tot}}=5.9\ (0.333\ \text{m/s})=1.96$ m/s, or a 21.6% reduction.

From Fig. 4 it is apparent that the maximum benefits regarding fuel consumption by extending the rendezvous time occur in the neighborhood of the 80- and 260-deg initial azimuth positions. For the example considered occurring at approximately 80-deg initial azimuth angle, the total $\Delta\hat{V}$ required drops from 2.1 m/s to 0.50 m/s, a 72% reduction.

Error Analysis

Figures 5 and 6 are polar plots of the terminal error for various combinations of azimuth angle and transfer angle. Figure 5 shows the error in the terminal position for a unit error (1 rad) in the direction of ΔV with respect to the line of sight ($\Delta\alpha$) or the error resulting from a unit error in estimated azimuth angle ($\Delta\gamma_0$). Figure 6 shows the error in the terminal position for a unit error in nondimensional initial velocity $\Delta\hat{V}$.

From Fig. 5 it is apparent that the terminal error resulting from an azimuth or ΔV orientation error depends very little on the nominal azimuth angle and increases with rendezvous time. As an illustration of this fact and the use of Fig. 3, consider a transfer angle of 0.8 rad and an error of slightly over 1 deg or 0.02 rad in ΔV orientation. From Fig. 5 the error for a nominal $\gamma_0=100$ deg is an approximate minimum of about $1.25(0.02)=0.025\ r_0$ and for $\gamma_0=190$ deg is an approximate maximum of about $1.5(0.02)=0.03\ r_0$, which for an initial distance of 1000 m gives an error between 25 and 30 m. For a transfer angle of 1.2 rad, these figures increase to between 31 and 36 m.

Figure 6, on the other hand, shows a strong dependence of terminal error due to initial velocity error on initial azimuth. For angles from approximately 270 to 60 deg and from 90 to 240 deg the relative errors are quite large and increase with transfer angle. However, for initial azimuth angles in the neighborhood of 90 and 270 deg, the magnitude (and direction) of the terminal error is virtually independent of transfer angle.

To get an idea of the order of magnitude of these errors and to illustrate the use of Fig. 6 consider the situation where the astronaut is initially 1000 m away and wants to intercept in 1000 s giving a $\tilde{V}_{\text{av}}=1$m/s. For the sake of discussion, assume an orbit for which 1000 s corresponds to a τ^* of 0.8. For an initial azimuth angle of 90 or 270 deg, an error of 10% or $\Delta\hat{V}=0.1$ would correspond to a terminal error (in the y direction only) of $0.9(0.1)\ r_0=0.09(1000)=90$ m. Note that for the same conditions and a τ^* of 2.4, this error grows only to 100 m. The corresponding errors for an initial azimuth of 0 or 180 deg are 122 m for $\tau^*=0.8$ and 270 m for $\tau^*=2.4$.

It should be noted that at the azimuth angles in the neighborhood of 0 and 180 deg, where the terminal error is most sensitive to initial velocity errors, the intercept chart, Fig. 2, has the best resolution with respect to required initial velocity.

Figures 7-9 show the effects of errors in initial conditions ($\gamma_0,\alpha,\hat{V}_0$) on the terminal velocity. It should be noted that these figures present the errors away from the nominal value of the terminal velocity and not the actual value of it. The values given in Figs. 7 and 9 are for a unit error of 1 rad in γ_0 and α, respectively, and those in Fig. 9 are for a unit error in $\hat{V}_0$.

A short example will illustrate the use of these figures and provide an order of magnitude for the errors. Using the conditions of the previous examples, $\tilde{r}_0=1000$ m, $t^*=1000$ s, $\tilde{V}_{\text{av}}=1$, and $\tau^*=0.8$ rad, if the normal initial azimuth angle, $\gamma_0=100$ deg, and an error of 0.02 rad exists in γ_0, then from Fig. 7, $\Delta\hat{V}_x{}^*=2.4$ and $\Delta\hat{V}_y{}^*=-1.7$. Hence, $\Delta V_x{}^*=2.4$ $(0.02)(1)=0.048$ m/s and $\Delta V_y{}^*=-1.7\ (0.02)\ (1)=-0.034$ m/s. If for the same $\tilde{r}_0$, $t^*=3000$ s, then $\tau^*=2.4$, $\tilde{V}_{\text{av}}=0.333$, and the errors become $\Delta V_x{}^*=0.007$ m/s and $V_y{}^*=-0.052$ m/s. Errors in α are dealt with in the same manner using Fig. 8. For the same conditions, errors in α cause a slightly smaller error in the terminal velocity than for corresponding errors in γ_0.

Finally, terminal velocity errors due to initial velocity errors can be determined from Fig. 9. For this example, with $t^*=1000$ s, an error of 0.1 m/s would give a $\Delta\hat{V}_0=0.1$. For $\gamma_0=100$ deg the terminal velocity errors are given by $\Delta V_x{}^*=-0.26\ (0.1)\ (1.0)=-0.026$ m/s and $\Delta V_y{}^*=-1.05$ $(0.1)\ (1.0)=-0.105$ m/s. The corresponding values for $t^*=3000$ s ($r_0=1000$ m, $\tilde{V}_{\text{av}}=0.333$ m/s) are obtained by noting the error $\Delta\hat{V}_0=0.1/0.33=0.3$. Hence, $\Delta V_x{}^*=-0.2$ $(0.3)\ (0.333)=-0.020$ m/s and $\Delta V_y{}^*=3.4\ (0.3)\ (0.333)=$ -0.340 m/s.

Conclusions

A chart (Fig. 2) has been developed which would aid an astronaut to perform an unaided intercept with a vehicle in a nominally circular orbit. This chart is universal in the sense that it is valid for all circular orbit radii and for all intercept times (transfer angles). For given initial position, azimuth angle, and range, the chart provides direct information as to magnitude and direction, measured from line of sight, for the initial velocity increment required for intercept. Furthermore, the resolution of the initial velocity is best for those initial positions where the terminal error is most sensitive to initial veloity error.

Additional charts (Figs. 3 and 4) have been developed which would aid the astronaut in planning and completing a rendezvous. The total velocity impulse chart (Fig. 4) can be used to select rendezvous times if fuel consumption is a critical issue. The terminal velocity chart (Fig. 3) provides the velocity impulse needed to complete the rendezvous at zero relative speed.

Simple error considerations can be performed using the charts given in Figs. 5-9. The effects of errors in initial azimuth, initial velocity direction, and initial velocity magnitude on terminal position and velocity are easily established. Such studies emphasize the accuracies required for performing the intercept maneuver.

Acknowledgments

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